

Scattering of sine-Gordon Breathers on a Potential Well

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Abstract

We analyse the scattering of sine-Gordon breathers on a square potential well. We show that the scattering process depends not only on the vibration frequency of the breather and its incoming speed but also on its phase as well as the depth and width of the well. We show that the breather can pass through the well and exit with a speed different, sometime larger, from the initial one. It can also be trapped and very slowly decay inside the well or bounce out of the well and go back to where it came from. We also show that the breather can split into a kink and an anti-kink pair when it hits the well.

1 Introduction

The sine-Gordon model is probably the most studied integrable model. One of the reasons for this is that it describes a large variety of physical systems ranging from the Josephson effects [1], particle physics [2], information transport in microtubules [3], non-linear optics [4], and ferromagnets [5].

From a mathematical point of view it is interesting for several reasons. First of all it is Lorentz invariant. This means that any stationary solution can be boosted to any speed, a key property to perform any scattering. Moreover as any finite energy solution, say, describing a kink, of the sine-Gordon model corresponds to a mapping from the circle into itself, each solution is characterised by a topological charge taking integer values. By

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conventions, solitonic solutions with a negative topological charge are called anti-kinks. In principle, kink and anti-kink could annihilate with each other, but because of the integrability of the model, instead, they scatter or form bound states which are called breathers. Breathers have been extensively studied and they are known, for example, to scatter with each other elastically, like kinks or anti-kinks.

In the inhomogeneous version of the sine-Gordon model, the coefficient in front of the potential becomes a function of x . In this paper we consider a square well potential where the potential coefficient is one everywhere except in a small region of finite width where it takes a smaller value. In physical applications the potential coefficient is usually determined by one of the physical properties of the system. It can be the magnitude of a magnetic field or an other property of the considered material. The square well we have chosen thus correspond to a system where those physical properties take two different values and where the transition between these values is very short compared to the size of the inhomogeneity and the size of the sine-Gordon kink or breather.

In a recent work [6], we have studied the scattering of a kink on the square well potential and now we investigate how the breather scatters on the same well. The scattering of a breather in inhomogeneous systems is not new. A few years ago F. Zhang[7] studied the scattering of the breather on a $1/\cosh^2$ potential. For very narrow well, the square well is very similar to the potential studied by him, while for a wide well, it is quite different, as in our case the non-integrability is generated by only two points, the edges of the well. This means that breathers and kinks can freely propagate inside a well that is larger than their own size. This, as we will show, has a large influence on the scattering properties of the breather.

In what follows, we present a detailed study of the scattering of the breather on a square well, analysing the dependence on all the parameters which influence this scattering. As Zhang, we have observed several modes of scattering: transmission or reflection of the breather, trapping of the breather and splitting of the breather into a kink and an anti-kink. We find a strong dependence of the scattering mode on the breather oscillation phase and then discuss the relative occurrence of these different modes when one varies the parameters of the model.

2 Sine-Gordon Model with a Potential Well

The sine-Gordon model with a square potential well is defined by the following Lagrangian

$$\mathcal{L} = \int \frac{1}{2} (f_t^2 - f_x^2 - 2k(1 - \alpha)(1 - \cos(f))) dx. \quad (1)$$

where

$$\alpha = a \quad -L/2 < x < L/2 \quad (2)$$

$$\alpha = 0 \quad \text{elsewhere.} \quad (3)$$

When a is negative, the potential is thus a square well of width L and depth $|a|$. The equation of motion is given by

$$f_{tt} - f_{xx} + k(1 - \alpha)f = 0. \quad (4)$$

The scattering of a kink on a square well potential was studied in [6] where it was shown that at small speeds, the kink becomes trapped by the well while at large speeds it goes through the well but loses some energy through radiation and thus exits from the well with a speed smaller than the initial one. The speed above which the kink can escape from the well was called the critical velocity.

For velocities in a few, very narrow, ranges of incoming speeds, just below the critical velocity, it was observed that the kink does neither go through the well nor get trapped in it, but instead bounces out of the well and returns to where it came from. Thus we have a reflection.

After studying the scattering of a kink on the square well, it is natural to ask what happens to a breather sent towards a similar well.

A breather which moves at speed v is described by [8]

$$f(x, t) = 4 \operatorname{atan}\left(\frac{\sin(\omega \sqrt{k}T) \sqrt{1 - \omega^2}}{\omega \cosh(\sqrt{1 - \omega^2} \sqrt{k}X)}\right) \quad (5)$$

where $X = \frac{x - vt}{\sqrt{1 - v^2}}$ and $T = \frac{t - vx}{\sqrt{1 - v^2}}$. This $f(x, t)$ is a solution of the pure sine-Gordon equation, *i.e.* (4) when $\alpha = 0$.

The energy of the breather can be easily calculated and is given by $E = 16 \sqrt{k} \frac{\sqrt{1 - \omega^2}}{\sqrt{1 - v^2}}$. Notice that in our units, the energy of a kink is equal to 8. It is well known that the breather is a bound state of a kink and an anti-kink and thus the energy of the breather is less than the energy of a kink and an anti-kink infinitely separated (*i.e.* 16).

A stationary breather is a periodic function in time of period $T = 2\pi/(\omega k)$. For small values of ω , the period of the breather is thus very large and, at the apex of the oscillations, the kink and the anti-kink are well separated. When ω is nearly 1, the period is slightly larger than $2\pi/k$, the maximum amplitude for the breather is small and the kink and anti-kink never really separate from each other.

The scattering of a breather on the well depends on several parameters: the breather parameter ω , the incoming speed v , the width L and the depth a of the well as well as the phase of the breather when it hits the well. As the breather is an extended object, the scattering time cannot really be defined precisely and so it is difficult to accurately determine the phase of the breather at the time of the scattering. Nevertheless it is straightforward to show that, in the well frame, the distance travelled by the breather outside the well, *i.e.* when $k = 1$, during one period of oscillation is equal to $d = 2\pi/(\omega \sqrt{1 - v^2})$. To cover the full set of breather phases all we have to do is to put the breather initially at several positions within the range $[-x_1 - d, -x_1]$ where x_1 must be

sufficiently far away from the edge of the well so that, initially, the breather does not overlap with it.

To investigate the dependence of the scattering on the parameters L , a , v and ω we have systematically scanned the full range of the breather phase by varying the initial position of the breather in the range $[-x_1 - d, -x_1]$ by step $dx = 0.02$ (we have thus used over 300 values of the phase for each parameter set). We have then counted the number of times each type of scattering has occurred and compared their relative occurrences.

Before we analyse the dependence of the scattering on these parameters we first describe the different phenomena that we have observed, that is, transmission, trapping, backwards and forwards scattering as well as backwards splitting. Initially, we consider the case of $v = \omega$ so that the energy of the breather equals the energy of a kink and an anti-kink. This is the critical case where the kink and anti-kink cannot split outside the well. So, unless otherwise stated, in the discussions below it is assumed that $v = \omega$. Later we report on what happens when $v \neq \omega$.

2.1 Breather Transmission

When a breather is sent on a well, one would expect that if the incoming speed is large enough or if the well is small and shallow, the breather should be able to go through it and emerge on the other side of the well. We have found that this is indeed the case, but the picture is more complicated than for the scattering of a kink on the well. We have found that the outcome of the scattering process is very sensitive to the phase of the breather. While at large speeds the breather seems to always pass through the well, it is also true that for most values of the well's width and depth, the soliton can also pass through the well for nearly any value of the speed if it has the right phase.

It is important to stress that the scattering is always inelastic. When the breather crosses the well it always radiates some energy away. The energy of the outgoing breather is thus always smaller than the initial energy. The radiated energy can come from two different sources: the kinetic energy of the breather or its internal energy. We have observed that indeed both energies can decrease but more surprisingly we have also seen that one can increase while the other one decreases, with the total, of course, decreasing. In particular, we have observed that sometimes the kinetic energy of the breather increase during the scattering. In these cases, the well thus acts like a slingshot. This is illustrated on Figure 1a where we present the position of the maximum of the energy density as a function of time. The oscillations are caused by the vibration of the breather and we clearly see that the speed of the breather jumps from $v = 0.1$, before the scattering, to $v = 0.17$ afterwards. The period of oscillation also changes from $T = 62.517$ to $T = 31.87$, corresponding to $\omega = 0.1$ and $\omega = 0.194$, respectively.

In Figure 1b we present the outgoing speed of the breather as a function the breather's initial position (*i.e.* its phase) for the case $L = 10$, $a = -0.2$ and $v = \omega = 0.1$. Notice that in this case the outgoing speed of the breather is nearly always larger than it initial speed. This is a generic feature we have observed for $v = \omega = 0.1$. When we took $v = \omega = 0.3$

we noticed that the outgoing speed tends to oscillate typically in the range $[0.2, 0.4]$. Looking at figure 1b, notice also the small amount of backwards scattering described below.

More surprisingly we have seen a few cases of a breather being split in two breathers (Fig 1c). One is ejected from the well while another one remains trapped inside the well. This leads us to another observed scattering outcome: the trapping of the breather.

2.2 Breather Trapping

When a breather scatters on a well, it can become trapped in it. As we will see later, this occurs more often when the well is deep. Once trapped the breather is actually unstable and it slowly radiates away its energy. This happens because of the perturbation introduced by the well.

As is well known [9], in the linear limit, a square well always has at least one vibration mode:

$$f(x) = \begin{cases} A \sin(\delta) \exp(-k \operatorname{ctan}(\delta)(x + L/2)) & x < -L/2 \\ A \sin(K(x + L/2) + \delta) & -L/2 \leq x \leq L/2 \\ A \sin(\delta) \exp(-k \operatorname{ctan}(\delta)(L/2 - x)) & L/2 < x \end{cases}, \quad (6)$$

where $\delta = \arcsin(\sqrt{1 + akL^2/4})$ and $K = \sqrt{-ak(1 + akL^2/4)}$. The period of oscillation of this vibration is given by

$$T = \frac{2\pi}{\sqrt{k(1 - a^2k\frac{L^2}{4})}} \quad (7)$$

and we see that, like the breather, the period is always larger than 2π . Note that the ground state obtained in the linear limit and the breather are very similar and can be thought of as sine-Gordon vibrations in the well derived in 2 different limits. The breathers are vacuum excitations of the sine-Gordon model. When confined in a large well they can have any width for as long as they fit inside the well. The amplitude of the breather is linked to its size and frequency. The linear vibrations in the well, on the other hand, have a fixed size and period, but their amplitude is arbitrary for as long as they remain small. In a relatively narrow well, the breather does not really fit inside the well and so becomes a large amplitude linear vibration.

The sine-Gordon vibrations in the well always decay by radiating away some energy: the *linear vibrations* radiate because of the nonlinearity of the sine-Gordon equation while the breather radiates because of the perturbation introduced by the well.

As stated in the previous section we have sometimes observed a breather going through the well but leaving a relatively large oscillation in it. In figure 1c the amplitude of oscillation for the excitation inside the well and the outgoing breather are 1.7 and 2.7 respectively (this is not visible in Figure 1c simply because the two oscillations are out of phase and we have chosen to plot the figure at the time when both excitations are relatively large). The formation of a double breather is very rare though; out of 100000 or so simulations that we have performed, we have only seen the creation of a double

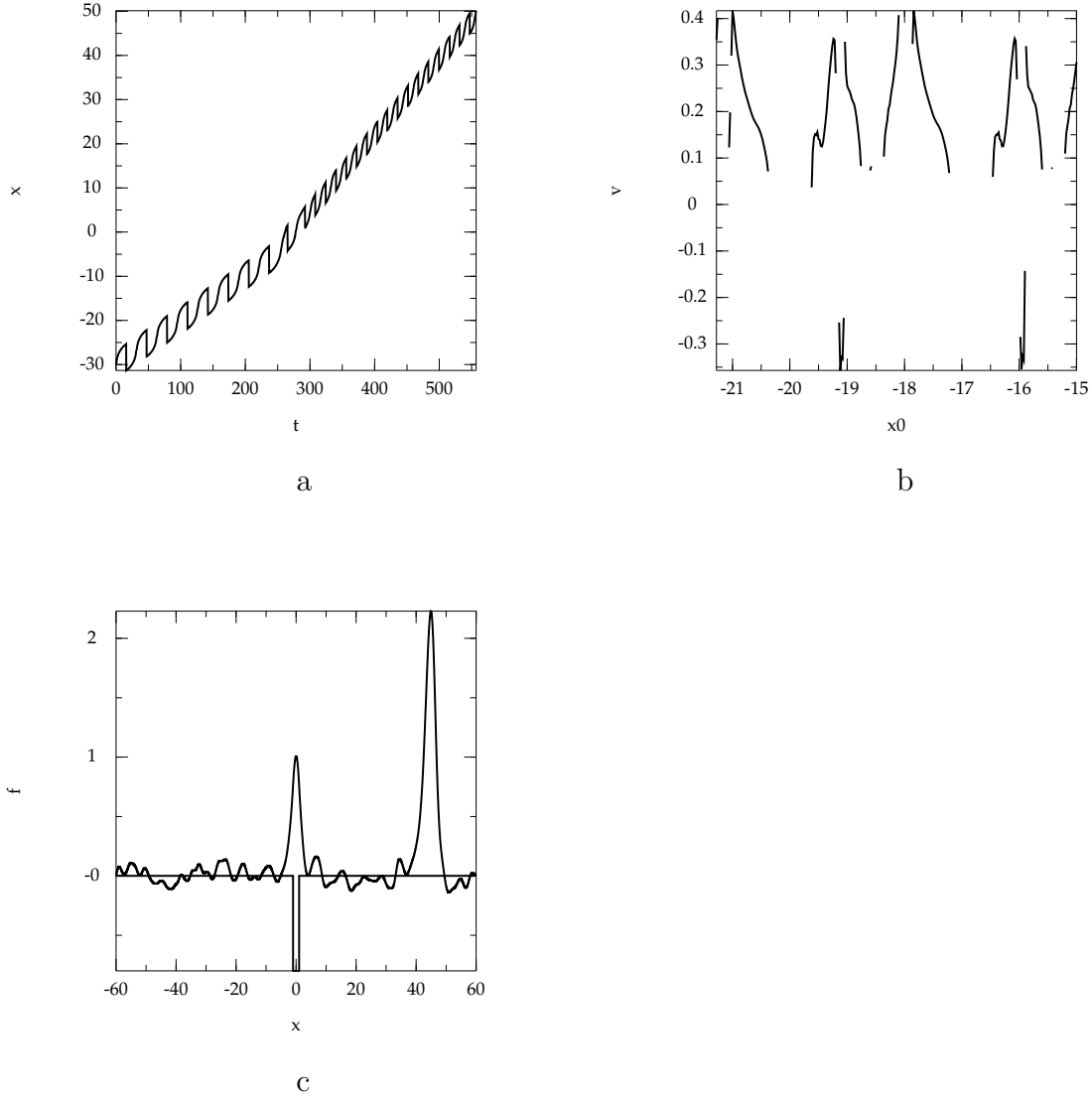


Figure 1: a) Breather position for a well with $L = 2$, $a = -0.2$, $v = \omega = 0.1$ and $x_0 = -29.92$. b) Breather outgoing speed for a well with $L = 10$, $a = -0.2$, $v = \omega = 0.1$. The gaps correspond to other types of scattering. c) A solution profile for $L = 2$, $a = -0.8$, $v = \omega = 0.3$ and $x_0 = -15.5$. The breather is split into two: one is ejected from the well while the other one is trapped inside it.

breather a few times in the regions of parameters corresponding to the transition between a breather trapping and a breather transmission.

2.3 Breather Backwards Scattering

The breather can sometimes bounce out of the well. As we will show later, this tends to occur mostly at small speeds and in a narrow and shallow well, but unlike what happens for the scattering for the kink, this scattering mode is observed for large ranges of the parameters values. As for the forwards scattering, the breather can be ejected backwards from the well with a speed larger than it had initially. This is seen in Figure 1b in two narrow regions of x_0 where the outgoing speed of the breather is negative.

In Figure 2 we present the time evolution of the position of the breather during a backward scattering for the case $L = 2$, $a = -0.2$, $v = \omega = 0.12$ and $x_0 = -11.6$. In this case the outgoing velocity is $v_{out} = -0.144$.

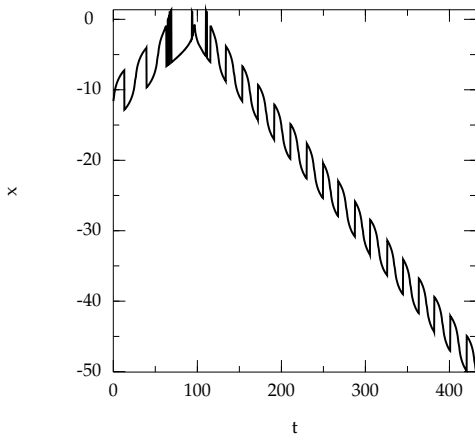


Figure 2: Position of the breather in the case of a well centred on $x = 0$ for $L = 2$, $a = -0.2$, $v = \omega = 0.12$ and $x_0 = -11.6$.

2.4 Breather Splitting

The most interesting phenomenon we have observed when scattering a breather on the well is the spitting of the breather into a kink and an anti-kink pair. As the energy of a breather or kink decreases when they fall into the well, the excess or energy can be used to split the breather into a kink anti-kink pair. One of them remains inside the well while the other escapes from it and moves backwards or forwards. Initially, we expected this

phenomenon to occur only in a very narrow region of the parameter space. It turns out, however, that, together with the transmission of a breather through the well, this is the most frequent outcome of the scattering process. The kink or anti-kink can be ejected from the well in either direction but, as will be shown later, the forward scattering is more frequent, especially for shallow and wide wells.

As the energy of a breather of frequency ω and speed v , outside the well, is $E_{br} = 16\sqrt{(1 - \omega^2)}/\sqrt{1 - v^2}$ and the energy of a kink trapped inside the well, assumed to be large enough to contain the kink, together with an anti-kink outside it is $E_{kak} = 8(1 + \sqrt{1 - a})$ we can easily evaluate the critical speed below which the splitting is impossible:

$$v_c = (1 - \frac{4(1 - \omega^2)}{(1 + \sqrt{1 - a})^2})^{\frac{1}{2}}. \quad (8)$$

When $v \leq \omega$, we find that $v > v_c$ and so we see that the splitting is always possible from an energetic point of view.

Notice that the trapping of a kink and the ejection of an anti-kink is equivalent to the trapping of an anti-kink and the ejection of a kink. To see this, we observe that if we multiply $f(x, t)$ by -1 , a kink becomes an anti-kink and vice-versa, while the breather becomes a breather with the opposite phase. So any solution with a trapped kink and an ejected anti-kink can be transformed into a solution with a trapped anti-kink and an ejected kink by changing the phase of the breather by 180 degrees.

The scattering of the breather on the well is inelastic and generates some radiation waves, so not all the trapping energy is transferred to the ejected kink (anti-kink). We have also observed that after the scattering, the kink and anti-kink wobble a little, radiating some energy away (the sine-Gordon kink does not have genuine vibration modes [10]). Moreover, the trapped kink (anti-kink) moves back and forth inside the well even when the well is quite narrow.

In Figure 3 we show the profile of a split breather after its scattering on a well with $L = 10$, $a = -0.2$, $v = \omega = 0.14$ and $x_0 = -17.12$. The outgoing speed of the ejected kink in this case is $v_{kink} = 0.109537$. The speed of the ejected kink or anti-kink varies with the phase of the breather. This is shown on Figure 3b where we present the outgoing speed of the kink after the scattering. Note that the speed of the outgoing kink (anti-kink) is nearly always larger than the incoming speed of the breather. This is true in most cases but it varies a little with the depth and the width of the well.

3 Parameter Dependence

The most important parameter in determining the properties of the scattering of the breather on the square well is the phase of the breather. In many instances, the scattering outcome is very sensitive to its value. This is shown in figure 4 where we present the scattering mode as a function of the breather phase (*i.e.* the breather initial position) in two extreme cases.

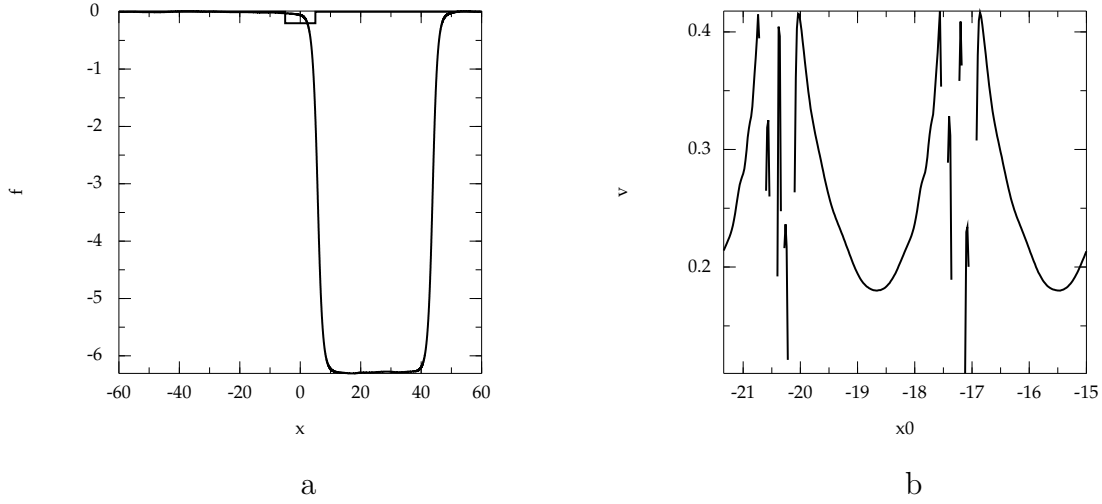


Figure 3: Split breather scattering with $L = 10$, $a = -0.2$, $v = \omega = 0.14$. a) Profile after the scattering for $x_0 = -17.12$. b) Outgoing speed of the kink or anti-kink (the gaps correspond to the regions of forwards scattering).

In Figure 4a, we have $a = 0.5$, $L = 20$ and $v = \omega = 0.1$, and we see that the scattering mode is very sensitive to the breather phase. This is very common for small values of v and ω , especially when most scattering modes can occur. The numbers in parenthesis below the mode names correspond to the total fraction of these modes for the full range of the phase.

In Figure 4b, we have $a = 0.2$, $L = 2.4$ and $v = \omega = 0.3$, and now the scattering is very regular: the breather splits into a kink or an anti-kink in two well defined phase regions and it scatters forwards in other cases. We have observed this type of pattern especially when the speed is large. Our explanation for this is that when the breather moves slowly, it has time to oscillate several times inside the well. Its vibration phase thus changes relatively rapidly during the scattering process, leading to different scattering modes. At large speeds, on the other hands, the breather mostly goes through the well and scatters with the phase it has at that time. This fits well with the observations we have made when studying the scattering of a baby-Skyrme soliton on a square well[11] where at low speeds, the soliton oscillates several times inside the well before emerging from it on one side of the well or the other.

3.1 Dependence on v and ω

By letting $v = \omega$, for the breather, its energy is exactly equal to the energy of a kink and an anti-kink infinitely separated. Thus this is the critical value for being able to generate a pair of kink and anti-kink outside the well and as such it is a natural choice of parameters to study the scattering modes of the breather. We will consider what happens

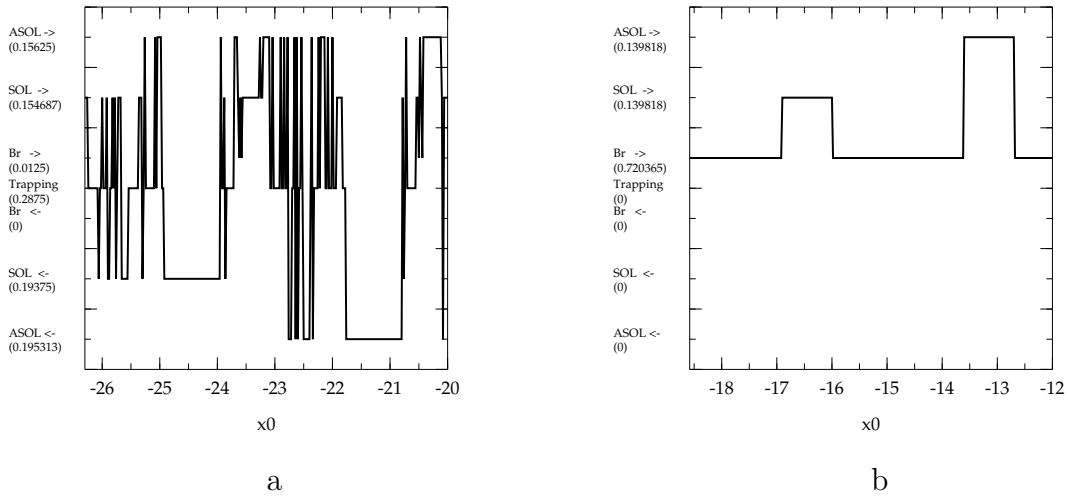


Figure 4: Phase dependence of the scattering modes for a breather a) $a = 0.5$, $L = 20$, $v = \omega = 0.1$; b) $a = 0.2$, $L = 2.4$, $v = \omega = 0.3$

when $v \neq \omega$ in a later section.

In figure 5, we present the relative occurrence of the different scattering mode as a function of the speed $v = \omega$ for two different wells: $a = 0.2$, $L = 2$ (fig 5a) and $a = 0.2$, $L = 10$ (fig 5b). For small values of v and ω , the breather are dominantly splits into a kink and a anti-kink with one of them trapped inside the well while the other escapes backwards. This is a general feature at low speeds when the well is narrow and reasonably deep. Looking at movies that we have made of several scattering of this type, we have always observed the following: when the breather hits the well, the kink (anti-kink) falls into the well and get trapped. Because of the narrowness of the well, the anti-kink (kink) is neither able to fall inside it nor to push the kink (anti-kink) outside the well. It has thus no other choice but to bounce on the trapped kink (anti-kink). If the initial speed is increased sufficiently, the second anti-kink (kink) has enough energy to push the trapped kink (anti-kink) at least partially out of the well. This can then result in a forwards or backwards scattering as well as in the splitting of the kink where a kink or an anti-kink is ejected forwards.

At large speeds, there are only two scattering modes: forwards transmission or forwards splitting. Note the oscillations between these two modes in figure 5b. At large v , there is always only one scattering mode, the transmission of the breather. This can be easily explained by the fact that the breather has enough energy to cross the well quickly without being affected much by it.

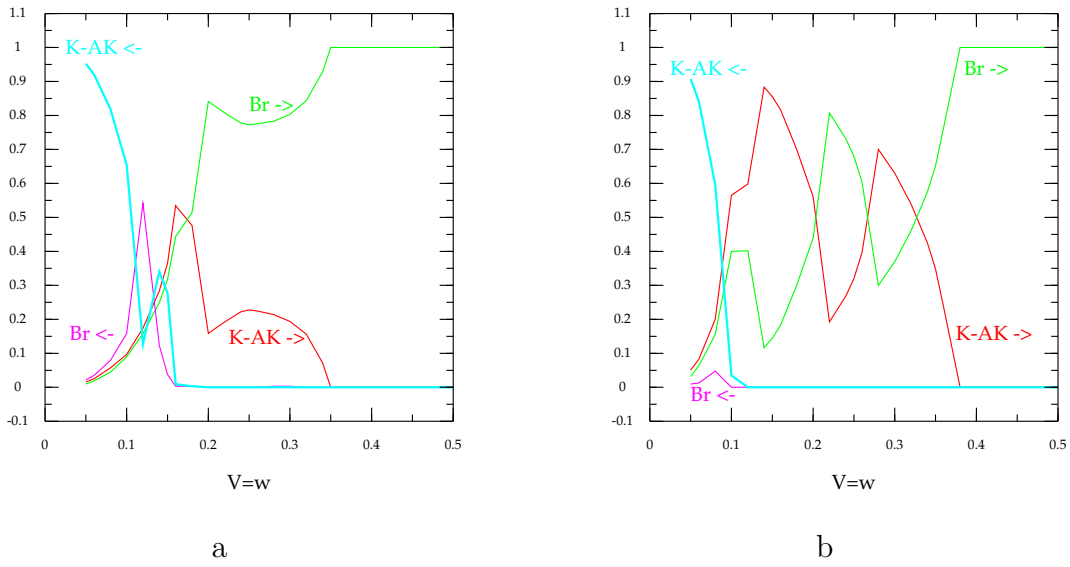


Figure 5: Scattering mode frequencies as a function of $v = \omega$. a) $a = 0.2$, $L = 2$; b) $a = 0.2$, $L = 10$. "Br- <" : forwards breather scattering. "Br < -" : backwards breather scattering. "K - AK- >" : trapped kink (anti-kink) and forwards anti-kink (kink). "K - AK- >" : trapped kink (anti-kink) and backward anti-kink (kink).

3.2 Dependence on the well width L

The width of the well also affects the scattering of the breather, as is shown in figure 6a for $a = 0.2$, $v = \omega = 0.1$ and in figure 6b for the cases $a = 0.2$, $v = \omega = 0.3$.

When $v = \omega = 0.1$ and for very narrow wells, the dominant scattering mode is the forwards splitting of the breather. The well is so narrow that the second kink has enough energy to push the first one out of the well. When the well is wider than 2 the dominant mode is the backwards splitting of the breather as explained in the previous section.

Once the well is larger than 10, only 2 scattering modes are relevant in the two cases presented: the forwards transmission and the forwards splitting. What we have observed when the well is very large is that when the breather enters the well its splits into a kink and an anti-kink pair. For a reason we have not understood yet, whichever of the kink or anti-kink is in front happens to have more energy than its partner. The kink and the anti-kink then move forwards, slowly increasing the distance separating them until one of them hits the other side of the well. As it has more energy, it manages to climb out of the well and to escape forwards while the second soliton, with less energy, remains trapped inside the well.

The phase of the breather when it falls into the well determines which of the kink or the anti-kink, into which the breather has divided, is at the front and eventually escapes from the well. In the intermediate region between the two modes, there is a succession of narrow windows where the breather goes through the well, separated by windows where

the kink of the anti-kink, into which the breather splits, escapes from the well.

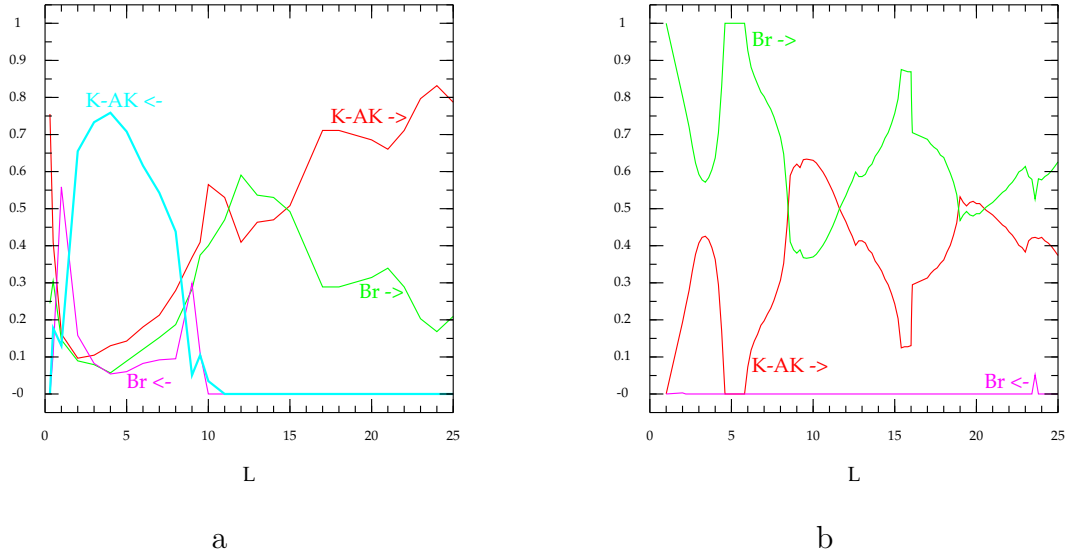


Figure 6: Scattering mode frequencies as a function of L . a) $a = 0.2$, $v = \omega = 0.1$; b) $a = 0.2$, $v = \omega = 0.3$

3.3 Dependence on the well depth a

The depth of the well plays a major role in the breather scattering as this is the parameter that determines the binding energy and the size of a breather or a kink inside the well.

As shown in figure 7 a, for a narrow well ($L = 2$) and at a small speed ($v = \omega = 0.1$), the scattering modes vary greatly with the depth of the well. For very shallow wells ($a < 0.02$), the dominant mode is the forwards transmission: the breather hardly sees the well at all. For marginally deeper wells ($0.02 < a < 0.15$), the dominant modes are the forwards splitting and then the backwards scattering. For deeper wells ($0.15 < a < 0.6$), as explained in a previous section, the backwards splitting dominates. Then when ($a > 0.6$) the well is so deep that the breather is usually trapped by the well.

When the speed is increased to $v = \omega = 0.3$, see figure 7b which looks very much like a stretched out version of figure 7a, except for the trapping curve. In this case, the breather has enough energy to go through the well quickly and the dominant mode is the forwards transmission until about $a > 0.6$ where the trapping dominates.

For wide wells, the picture changes significantly. The as explained in the previous section, the breather nearly always splits into a kink anti-kink pair inside the well. For the case presented in figure 7c, $L = 20$ and $v = \omega = 0.1$, the parameters are such that the forwards splitting is dominant for ($a < 0.45$). For deeper wells, the scattering mode changes very rapidly as a increases, but overall, the breather trapping dominates.

When the speed is increased to $v = \omega = 0.3$, see figure 7d, the two dominant modes are the forwards splitting and the forwards transmission. Trapping occurs rarely, only for very deep wells. In this case, the breather has enough energy to interact with the well quickly and, at least in part, escape from it.

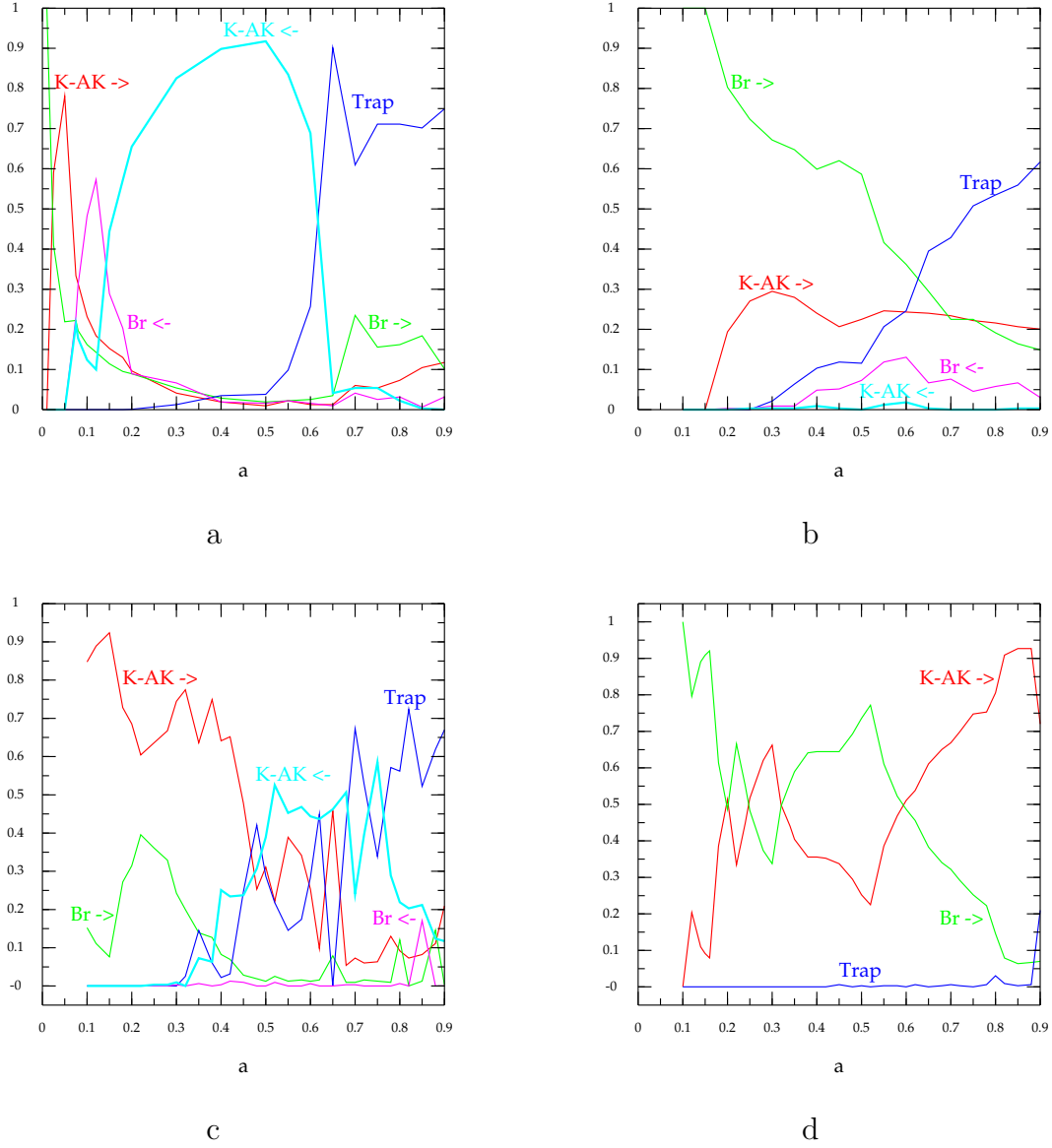


Figure 7: Scattering mode frequencies as a function of a . a) $L = 2, v = \omega = 0.1$; b) $L = 2, v = \omega = 0.3$; c) $L = 20, v = \omega = 0.1$; d) $L = 20, v = \omega = 0.3$;

4 Varying v and ω separately.

So far we have looked only at the scattering of a breather on a well for the special case $v = \omega$, that is when the breather has exactly the same energy as an infinitely separated

pair of a kink and an anti-kink. This critical case is particularly interesting, but it is also very interesting to investigate what happens when v and ω differ.

The results are shown of Fig 8 where we have taken $a = 0.2$ for the depth of the well and we have considered a well of widths $L = 10$ and $L = 20$ for $\omega = 0.1$ and $\omega = 0.3$.

Looking at the figures we note that when $v \gg \omega$, the breather is dominantly going through the well. In the three cases we have considered, the largest rates of kink anti-kink splitting occur in the region where $v \approx \omega$, the actual maximum being reached when v is slightly larger than ω .

When v is smaller than ω , all the familiar modes, *i.e.* trapping, backward scattering of the breather and backward kink anti-kink splitting, can all occur but at a relatively small rate. This can be explained by the fact that, having less kinetic energy, the breather is more likely to bounce on the far side of the well and, if the phase is right, come out of the well.

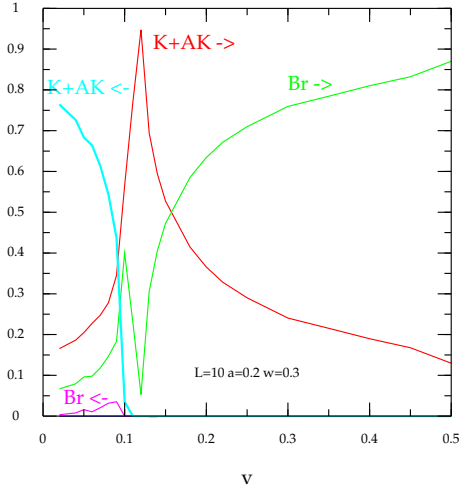
5 Conclusions

In this paper, we have shown that the scattering of a breather on a square well exhibits very interesting phenomena. The breaking of the sine-Gordon integrability due to this inhomogeneity leads to scattering modes that are forbidden in an integrable model. In particular the sine-Gordon breather can be split into a kink and an anti-kink which move both forwards and backwards. Somewhat surprisingly, this scattering mode is genuine and sometimes, depending on the parameters of the model, the dominant one.

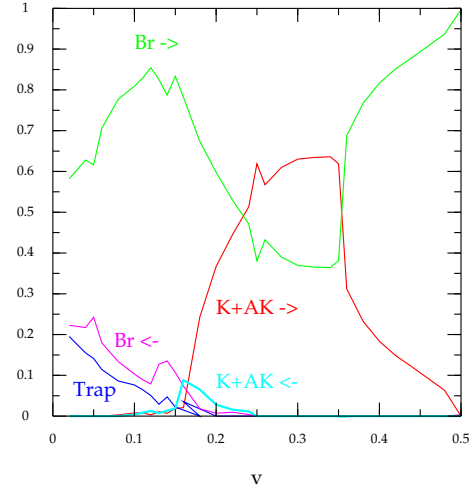
Another surprising phenomenon seen in the scattering is that the well can accelerate the breather. This is possible because the internal energy of the breather can be partly converted into kinetic energy. This acceleration can occur for the forwards as well as the backwards motion.

The parameter dependence of the scattering data is quite complicated. It is also very sensitive to the phase of the breather when it collides with the well. Overall, we have observed that at high energies, the breather tends to scatter forwards or to split forwards into a kink/anti-kink pair while at low energies on the other hand, all the scattering modes can take place.

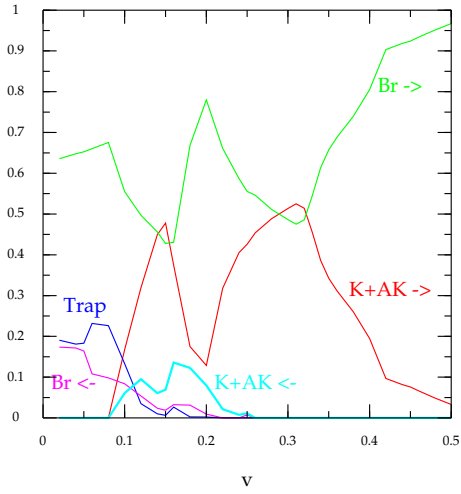
Given our observations, it would be interesting to find out if one can reverse the scattering process and create a breather by the scattering of a kink on a trapped anti-kink (or vice-versa). If this process is possible, then it would provide a method to experimentally generate a breather from a kink and an anti kink. We plan to investigate this in the future.



a



b



c

Figure 8: Scattering mode frequencies as a function of v for $a = 0.2$ and a) $L = 10$, $\omega = 0.1$; b) $L = 10$, $\omega = 0.3$; c) $L = 20$, $\omega = 0.3$;

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